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FRAGMENTATION OF A COMPOSITE MATERIAL AND FRAGMENTATION OF FIBRES UNDER A DYNAMIC LOAD*

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A simple model is proposed which qualitatively describes the phenomenon of the dynamic loading of a fibrous material. The model is based on a treatment of the laws of conservation of energy and momentum during the fragmentation of a material. A dependence of the sizes of the fragments on their shapes, rates of deformation, densities and energies of formation of new surfaces is introduced. By fragments, we mean those parts of bodies which have been formed during the break up of the latter. The fibre fragments of a composite are called splinters. A comparison is made with experimental data.

We shall mention a number of papers on the fragmentation of materials in which hypotheses have been introduced concerning the local conservation of energy and momentum. Some of the first papers in which the law of conservation of energy was used to represent the sizes of the fragments were /1, 2/. It was assumed that the whole of the kinetic energy of the material was transformed into energy for the formation of new surfaces during the break up of the material. This assumption leads to a substantial reduction in the sizes of the fragments, since the fraction of the kinetic energy which is carried away by the dispersing splinters was not taken into account. Different models were proposed in /3, 4/ which allow for a reduction in the fraction of the kinetic energy which transforms into the kinetic energy of the fragments. Account was also taken of the fact that fragmentation and crack propagation take place over a finite time. The problem of coarse fragmentation during homogeneous all-round expansion of an unbounded volume was considered in /5/ and it was found experimentally that, during the fracture of a material, energy is consumed in the formation of new surface, which is equal to the difference between the kinetic energy of the body before fragmentation and the energy of the fragments. Experimental confirmations of this hypothesis have been presented in /6, 7/ and it has been shown that the fraction of the energy which is accumulated due to elastic deformation is relatively small.

The specific features of the behaviour of composite materials under intense loads have been studied starting from /8-12/. It has been established that the principal factor which reduces the strength of a composite material in the zone of intense action is a fragmentation of the fibres which leads to a state of affairs where the fibres break up into splinters with sizes comparable with the critical length /13/. Below, a hypothesis on the energy balance during breakdown is put forward in order to describe the process of the breakup of fibres and the fragmentation of a composite material. The equations of motion of the fragmented material are introduced. The dependence of the size of the fragments on their shape and the rate of deformation field is established.

1. Local energy and momentum conservation zones. We will now formulate the basic hypotheses and derive the equations which describe the fracture of a continuous body by inertial forces. Let us visualize a continuous body, the particles of which move at high velocities. The velocity field of the particles is inhomogeneous and obeys the equation of continuity. In certain domains of the body, zones can arise where there are significant tensile stresses. The coupling forces between individual particles of the solid block the fracture process, and the fragments which are formed after the fracture of the solid therefore have finite dimensions.

An upper estimate of the magnitude of the characteristic size of the fragments can be obtained in the following manner /1, 2/. It is assumed that all the kinetic energy of the solid is expended in forming new surfaces. It can be shown that this assumption leads to an inversely proportional dependence of the sizes of the splinters on the rates of deformation.

Let us now determine the dependence of the sizes of the fragments on the velocity by estimating the fraction of the energy which is carried away by the fragments after the breakup of the solid. We will assume that a certain continuous solid Ω is decomposed into a set of rather small fragments and we will denote a certain typical splinter by ω and its surface after fragmentation by Γ. At the instant of time immediately preceding

Prikl. Matem.Mekhan.,54,4,695-699,1990 fragmentation, the fragment ω belongs to the continuous body. Following /14/, we write down the laws of conservation of the energy and momentum of the splinter in the form of integral identities. We will denote by $\omega \times T$ the four-dimensional volume which describes the motion of the fragment ω in the space (X_1, X_2, X_3, t), where X_1, X_2, and X_3 are the spatial coordinates and t is the time. The law of conservation of momentum for an arbitrary four-dimensional volume has the form

$$\iiint\limits_{\partial(\omega\times T)} \rho u_i dX_1 dX_2 dX_3 + (\rho u_i u_1 - \sigma_{i1})\, dX_2 dX_3 dt + (\rho u_i u_2 - \sigma_{i2})\, dX_3 dX_1 dt + (\rho u_i u_3 - \sigma_{i3})\, dX_1 dX_2 dt = 0 \tag{1.1}$$

where ρ is the density of the material, $\partial(\omega \times T)$ is the surface of the four-dimensional volume, $u_i = u_i(X_1, X_2, X_3, t)$ are the components of the velocity vector of the fragments, and σ_{ij} are the components of the stress tensor. The Latin subscripts take the values 1, 2, and 3, while summation is carried out over doubly repeating indices from 1 to 3.

Let R_0 be the radius vector of the centre of mass of a fragment at the instant preceding fracture and let U_i be the velocity components of the centre of mass of the fragment. The components of the velocity vector u_i at a point with a radius vector R belonging to a fragment ω can be represented in the form of a Taylor's series:

$$u_i(R) = U_i(R_0) + x_i e_{ij}^{\cdot}(R_0) + \ldots \tag{1.2}$$

where x_i are the components of the vector $r = R - R_0$ and $e_{ij}^{\cdot} = {}^1/_2 (u_{i,j} + u_{j,i})$ are the components of the rate of deformation tensor.

We substitute (1.2) into the integral identity (1.1) and use Gauss's theorem:

$$\iiiint\limits_{\omega\times T} \left[\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j - \sigma_{ij})\right] dt\, dx_1\, dx_2\, dx_3 = \tag{1.3}$$

$$\iiiint\limits_{\omega\times T} \Big[\frac{\partial}{\partial t}(\rho U_i) + \frac{\partial}{\partial t}(\rho e_{ij}^{\cdot} x_j) + \frac{\partial}{\partial x_j}(\rho U_i U_j + \rho U_i e_{ij}^{\cdot} x_j + \rho e_{il}^{\cdot} e_{jk}^{\cdot} x_l x_k - \sigma_{ij}) + o(x_i x_j)\Big]\, dt\, dx_1\, dx_2\, dx_3, \quad (dt\, dX_1\, dX_2\, dX_3 = dt\, dx_1\, dx_2\, dx_3)$$

($o(x_i x_j)$ are terms containing derivatives of the components of the vectors x_i of the third and higher orders of smallness). Since the fragments are assumed to be fairly small, these terms will be neglected. Terms containing the linear forms of x_i are zero: $\iiiint\limits_{\omega\times T} \rho x_i dx_1 dx_2 dx_3 dt = 0$, since U_i are the components of the velocity vector of the centres of mass of the fragments. By assuming that ρ is a constant quantity, we obtain from (1.3):

$$\rho \frac{\partial U_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho U_i U_j + s_{ij} - \sigma_{ij}) = 0 \tag{1.4}$$

$$s_{ij} = \rho I_{lk} e_{il}^{\cdot} e_{jk}^{\cdot}/m, \quad I_{ij} = \iiint\limits_{\omega} \rho x_i x_j dx_1\, dx_2\, dx_3$$

$$m = \iiint\limits_{\omega} \rho dx_1\, dx_2\, dx_3$$

The quantity m is the mass of the fragment, I_{ij} are the components of the moment of inertia tensor of the fragment, and s_{ij} is the tensor describing the stresses which arise within the fragment and are due to the presence of inertial forces.

The law of conservation of energy for an arbitrary four-dimensional volume when there are no mass forces, internal heat sources and heat transfer has the form /14/

$$\iiiint\limits_{\omega\times T} \left\{\frac{\partial}{\partial t}\left[\rho\left(E + \frac{1}{2} u_i u_i\right) \frac{\partial}{\partial x_j}\right]\left[\rho u_j\left(E + \frac{1}{2} u_i u_i\right) - u_k \sigma_{kj}\right]\right\} dx_1\, dx_2\, dx_3\, dt = 0 \tag{1.5}$$

where E is the internal energy per unit mass.

By substituting the series (1.2) into the identity (1.5) and using the equation for the momenta (1.4) and the continuity equation $\partial U_j / \partial x_j = 0$, we get

$$\frac{d}{dt}\left(\rho E + \frac{1}{2} s_{ii}\right) = (\sigma_{ij} - s_{ij}) \frac{\partial U_i}{\partial x_j} \tag{1.6}$$

$$d / dt = \partial / \partial t + U_j \partial / \partial x_j$$

The left-hand side of this expression is the rate of growth of the sum of the internal and kinetic energies per unit volume in the case of a fixed fragment, while the right-hand side is the sum of the power of the internal surface forces and the forces of inertia. The change in the internal energy on fragmentation is usually associated with the formation of new surfaces. In other words, a fraction of the kinetic energy $\frac{1}{2}s_{ii}$ becomes accessible for transformation into the energy of the formation of new surfaces.

Let us first consider the case of an isotropic material. The increase in the internal energy when a fragment ω with a surface are S is formed is equal to γS, where γ is the surface energy. The change in the internal energy referred to the mass of the fragment m, is equal to $\Delta E = -\gamma S/m$. Let us now assume that the fragmentation occurs instantaneously. In this case, the contribution from the operation of internal forces and inertial forces is negligibly small and the change in the sum of the internal and kinetic energies during fragmentation remains constant, whence

$$\gamma S = \frac{1}{2} I_{lk} e_{il}^{\cdot} e_{ik}^{\cdot} \tag{1.7}$$

In the case of anisotropic materials, the surface energy of fracture depends on the orientation of the surface. Let us consider the case of an orthotropic body. In an orthotropic solid the energies of formation of new surfaces with normals which are symmetrical about the planes of symmetry are identical. Let a small new surface with an area dS be formed within the body. The surface energy depends on the orientation of the unit normal to the surface, n, with respect to the triplet of vectors of the normals to the planes of symmetry i_1, i_2 and i_3 and is proportional to the area dS:

$$dE = \gamma(\alpha_k)\, dS, \quad \alpha_k = n \cdot i_k \tag{1.8}$$

(α_k are the components of the vector n in the basis i_1, i_2 and i_3 and $\gamma(\alpha_k)$ is a certain function). Hence, the equality which expresses the constancy of the sum of the surface and kinetic energies reduces to the form:

$$\int_S \gamma(\alpha_k)\, dS = \frac{1}{2} I_{lk} e_{ik} e_{il} \tag{1.9}$$

2. ***Sizes and shapes of the fragments.*** The determination of the sizes of the fragments in the case of isotropic deformation will be the first application of the formulae derived in Sect.1.

Let us consider a thin-walled sphere of thickness h, to the internal surface of which a pressure is applied at a certain instant of time. Let the velocity of the particles in the radial direction become equal to V. In a spherical system of coordinates, the components of the rate of deformation are equal to $e_\theta^{\cdot} = e_\varphi^{\cdot} = V/R$. Let the sphere break up with the formation of a number of fragments of regular shape. If the fragments have the shape of spherical segments then, for these fragments, $I_{\varphi\varphi} = I_{\theta\theta} = \pi L^4 h/32$, and $S = \pi L h$, where L is the diameter of the segment. By substituting this expression into (1.7) and solving for the diameter L, we get

$$L = c\,(\gamma R^2/(\rho V^2))^{1/3} \tag{2.1}$$
$$c = 16^{1/3} \approx 2{,}51 \ldots$$

We note that, if the shape of the fragment is taken to be square, the length of a side of the square is determined using (2.1) in which one should put $c = 12^{1/3} \approx 2{,}28 \ldots$.

The dependences which have been derived therefore enable one to determine the characteristic dimensions of the fragments if the material is deformed isotropically. If however, the rate of defromation depends on the direction, the formation of fragments of a regular shape whould not be expected. Relationships (1.7)-(1.9) are then inadequate both for determining the dimensions as well as the shape of the fragments.

Let us now study the question of the fragmentation of a thin plate under anisotropic deformation. Let $\varepsilon_x^{\cdot}$ and $\varepsilon_y^{\cdot}$ be the principal rates of deformation and their directions coincide with the x and y axes of a rectangular system of coordinates. We assume that the plate breaks up into a number of identical rectangular pieces during fragmentation. The directions of the sides of the rectangles are identical to the directions of the principal rates of deformation while the dimensions of the sides along the x and y axes are equal to a and b, respectively. We shall take the thickness of the plate as being equal to h. The area of the surface of the frament which has been formed is equal to $S = 2(a+b)h$ while the moments of inertia of the broken off chip along the x and y axes are equal to $I_{xx} = ha^3b/12$ and $I_{yy} = hab^3/12$. By substituting these quantities into (1.7) and solving the resulting equation for the surface area S, we get

$$S = h\left[\frac{96\gamma(1+\chi)^4}{\rho\varepsilon_x^{\cdot 2}(\chi^3+\lambda\chi)}\right]^{1/3}, \quad \lambda = \left(\frac{\varepsilon_x^{\cdot}}{\varepsilon_y^{\cdot}}\right)^2, \quad \chi = \frac{a}{b} \tag{2.2}$$

It can be seen that the surface energy γS depends on the ratio of the sides of the rectangles χ. Since the sum of the surface and kinetic energies of the fragments is equal to the kinetic energy of the body prior to fragmentation, the ratio of the kinetic energy of the fragments formed from the body to the kinetic energy of the body prior to fragmentation depends on the ratio of the sides χ. It follows from (2.2) that, during fragmentation, fragments with arbitrary ratios of the sides may appear and, therefore, the laws of conservation of energy and momentum alone do not enable the shape of the fragments to be determined uniquely.

Physically non-contradictory results can be obtained by invoking additional hypotheses. In fact, let us assume that the shape of the fragments which are formed are such that their kinetic energy, which is considered as a functional of the shape, is a maximum. When this is so, the fraction of the energy used in forming new surfaces will reach a minimum value.

As applied to the problem under consideration, this proposal reduces to the problem of finding the minimum of the function $S(\chi)$. It can be shown that it is attained for a ratio of the side $\chi^* = (\lambda-1)^{1/3}[(\sqrt{\lambda}-1)^{1/3}-(\sqrt{\lambda}+1)^{1/3}]+1$. As λ varies from 0 to ∞, the magnitude of χ^* falls off monotonically from 3 to $1/3$. Hence, if the deformation is close to uniaxial $(\lambda\to 0$ or $\lambda\to\infty)$, the size of the fragment in the direction of the deformation axis is twice the dimension along the axis in the direction of which the rate of defromation is equal to zero. In the case when the body experiences uniaxial deformation along the x axis at a rate of deformation $\varepsilon^{\cdot}$, the area of the surface of the fragments which has been formed is calculated using formula (12) in which one should put $\chi=3, \lambda=0$. By using the relationship $b=\chi a = S/[2h(1+\chi)]$, we find

$$a = 3c\left(\frac{\gamma}{\rho\varepsilon_2^{\cdot}}\right)^{1/3}, \quad b = c\left(\frac{\gamma}{\rho\varepsilon_2^{\cdot}}\right)^{1/3}, \quad c = \left(\frac{16}{9}\right)^{1/3} \approx 1{,}21\ldots$$

Similar results are also obtained in other cases if it is assumed that fragments with regular shapes are formed (hexagonal or elliptical fragments, for example).

3. *The fragmentation of the fibres of a composite material under a high rate of loading.*

We will use the relations which have been derived above to describe the fragmentation of the fibres of a composite material under a high rate of loading. It was found in experiments on the impact loading of a boron-aluminium composite material that fracturing of the fibres and exfoliation of the matrix occurred independently and, in fact, fracture of the fibres arose during the passage of a compression shock wave, and these fractures were subsequently accompanied by the separation of the matrix layers. It has actually been shown in /10/ that fibre fractures have the greatest effect on the reduction in the strength of composite materials under the action of intense loads.

Let us now compare the dimensions of fibre splinters determined using formula (1.7) and those obtained experimentally /10/. In a series of experiments boron-aluminium samples were subjected to the action of a Mylar plate with a thickness of 2.54×10^{-4} m in such a way that the pulse duration was 0.2×10^{-6} s. The rear surface of the samples was supported on a massive aluminium plate which ensured the passage of the compression wave without reflection from the end surface. The samples consisted of a packet of orthogonally stacked monolayers. The latter were formed by continuous fibres with a thickness $d=1.09\times10^{-4}$ m. The boron fibres had the following values of the physical constants: density $\rho=2.58\times10^3$ kg/m^2, surface energy $\gamma=50$ J/m^2, Young's modulus $E=4.21\times10^{11}$ Pa and longitudinal velocity of sound $c=12.1\times10^3$ m/s. The following experimental data /10/ are also presented: the maximum pressure in the shock wave p_{max} and the velocity of motion of the impacter U.

The following technique is proposed for calculating the dimensions of the splinters. The maximum deformation $e_{max}=p_{max}/E$ is calculated using the data on the maximum pressure in the wave. The deformation increases from zero up to a maximum value e_{max} over a time of the order of that required for the shock wave to pass through the transverse cross-section of the fibre $\tau=d/c=0.82\times10^{-8}$ s. An estimate of the rate of deformation $\varepsilon^{\cdot}=e_{max}/\tau$ is obtained from this. The use of formula (1.8) yields the estimate

$$L = \left(\frac{24\gamma\tau^2E^2}{\rho p_{max}^2}\right)^{1/3} \tag{3.1}$$

for the size of splinter.

The experimental data and the dimensions of the splinters calculated using these data are shown in Table 1.

Table 1

U, km/s	$p_{max} \times 10^{-8}$ Pa	$\varepsilon \cdot \times 10^{-7}$	$L \times 10^4$, m	
			Theory	Experiment
0.73	52	0,063	1,05	—
1,39	52	0.15	0,59	0.88±0.4
2.40	115	0.33	0,35	—
0.69	23	0.066	1.02	—
1.20	43	0.124	0.67	—
2.40	113	0.327	0.35	0.73±0.4

Formula (3.1) predicts somewhat lower dimensions for the splinters compared with those observed experimentally. This is possibly associated with the existence of other mechanisms for the dissipation of kinetic energy, such as viscoelastic deformation, plastic flow of the matrix and absorption of energy during the fracture of a fibre- matrix bond.

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